THERMAL CONDITIONS IN COUETTE FLOW OF A VISCOUS LIQUID

Zh. A. Zinenko, A. M. Stolin, and F. A. Khrisostomov

UDC 532.135

In experimental examples of hydrodynamic thermal breakdown (HTB) [1], it has been observed that it is possible to use this phenomenon to determine viscosity as a continuous function of temperature over a wide range using a nonisothermal rotational viscometer [2]. If the liquid is heated uniformly with time, i.e., a zero-gradient thermal process holds, and quasistationary hydrodynamic flow conditions obtain, then the reduction of the viscometer data is appreciably simplified, and the computation scheme suggested in [1, 3] is valid. Nonisothermal Couette flow between two cylinders has been studied under conditions with constant rotation rate, and hydrodynamic thermal breakdown is not observed [4-6].

§1. We consider Couette flow of a viscous incompressible liquid, located between two coaxial infinite cylinders, of which the inner, of radius $r = r_0$, rotates under a given shear stress $\sigma_g = \sigma_{r\varphi}(r_0)$, and the outer, of radius $r = r_1$, is fixed. We assume that the viscosity is an exponential function of temperature [7]

$$\mu(T) = \mu_0 \exp [U/(RT)], \tag{1.1}$$

where μ_0 and U are constant; R is the gas constant; and T is the absolute temperature.

The unsteady system of equations describing the motion and the thermal balance, accounting for energy dissipation, and the rheological equation for a Newtonian liquid can be written in the form

$$\frac{\partial \omega_\varphi}{\partial t} = \frac{1}{\rho r^3}\frac{\partial}{\partial r}(\sigma_{r\varphi} r^2), \tag{1.2}$$

$$c\rho \frac{\partial T}{\partial t} = \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{\sigma_{r\varphi}^2}{I\mu(T)},$$

$$\sigma_{r\varphi} = \mu(T)\, r \frac{\partial \omega_\varphi}{\partial r},$$

where ω_φ is the angular rate of rotation; t is the time; ρ is the density; λ is the thermal conductivity of the liquid; C is the heat capacity; $\sigma_{r\varphi}$ is the shear stress; r is the radius; and I is the mechanical equivalent of heat.

We assume that at zero time the liquid temperature is equal to that of the surrounding medium, while the stress profile corresponds to isothermal steady flow

$$T(r) = T_0,\ \sigma_{r\varphi}(r) = -\sigma_g (r_0/r)^2 \quad \text{at} \quad t = 0. \tag{1.3}$$

To heat the test material with viscous flow in a nonisothermal viscometer, the internal cylinder is made thin walled and hollow, while the outer cylinder is made with double walls, with the space between filled with a heat-insulating material [2]. In this case the inner cylinder is heated slowly by dissipative heating, while heat transfer between the outer cylinder and the surrounding medium follows Newton's law. Then the boundary conditions can be written in the form

$$\lambda S_I (\partial T/\partial r) = C_м \rho_м V_м (\partial T/\partial t) \quad \text{at} \quad r = r_0; \tag{1.4}$$

$$-\lambda(\partial T/\partial r) = \alpha(T - T_0) \quad \text{at} \quad r = r_1,$$

Chernogolovka. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 103-109, May-June, 1977. Original article submitted May 24, 1976.

where S_l is the lateral surface for heat transfer between the liquid and the inner cylinder; C_M, ρ_M are the heat capacity and the density of the material of which the inner cylinder is made; and V_M is the volume of the cylinder.

Let the angle φ increase clockwise and let the inner cylinder rotate in a positive direction; then the stress on the inner cylinder will be negative, and therefore

$$\sigma_{r\varphi} = -\sigma_g\,(\sigma_g > 0) \quad \text{at} \quad r = r_0; \qquad (1.5)$$
$$\omega_\varphi = 0 \quad \text{at} \quad r = r_1.$$

Converting Eqs. (1.2)-(1.5) to dimensionless form, we obtain

$$\frac{1}{\Pr}\frac{\partial\omega}{\partial\tau} = \frac{1}{x^3}\frac{\partial}{\partial x}(\sigma x^2), \qquad (1.6)$$
$$\frac{\partial\Theta}{\partial\tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial\Theta}{\partial x}\right) + \delta\frac{\sigma^2}{s^4}e^{\Theta/(1+\beta\Theta)},$$
$$\sigma = e^{-\Theta/(1+\beta\Theta)}x\frac{\partial\omega}{\partial x};$$
$$\Theta = 0,\ \sigma = -(s/x)^2 \quad \text{at} \quad \tau = 0,$$
$$F(\partial\Theta/\partial x) = \partial\Theta/\partial\tau, \quad \sigma = -1 \quad \text{at} \quad x = s,$$
$$\partial\Theta/\partial x = -\mathrm{Bi}\Theta, \quad \omega = 0 \quad \text{at} \quad x = 1,$$

where

$$\Theta = (T - T_0)\,U/(RT_0^2), \quad \sigma = \sigma_{r\varphi}/\sigma_g,$$
$$\omega = \omega_\varphi\mu(T_0)/\sigma_g, \quad x = r/r_1, \quad \tau = t\lambda/(C\rho r_1^2)$$

are dimensionless variables, and

$$\delta = \sigma_g^2 r_0^4 U/(\lambda\mu(T_0)\,I r_1^2 R T_0^2), \quad \beta = RT_0/U,$$
$$\Pr = C\mu(T_0)/\lambda, \quad \mathrm{Bi} = \alpha r_1/\lambda, \quad F = C\rho r_1 S_l/(C_M\rho_M V_M)$$

are dimensionless parameters.

The system of equations was solved numerically on a computer. The functions $\Theta(x,\tau)$, $\omega(x,\tau)$, and $\sigma(x,\tau)$ were determined for various values of the parameters δ, and Bi, F, s, Pr.

§2. The HTB phenomenon can arise [1, 3] in nonisothermal dissipative flow of a liquid in a rotational viscometer model with constant shear stress σ_g. Hence it follows that, for values of the thermal parameter δ greater than the critical value ($\delta > \delta^*$), the system of equations (1.6) does not have a steady solution, giving the distribution of temperatures and velocities; in fact, these increase progressively with time. The relation between the critical value δ^* and the geometric parameter s was determined analytically in [8] by solving the system (1.6) in the steady case for particular values of the parameters F and Bi. Below we study some special features in the development of HTB with time. Figure 1a-d shows the distributions of angular velocity and temperature (dashed and solid lines, respectively) with radii at various times τ for limiting cases of the thermal boundary conditions. The values of δ were chosen to lie above the breakdown limit ($\delta > \delta^*$).

1. Isothermal Cylinders: $F = 0$, $\mathrm{Bi} = \infty$ (Fig. 1a). The temperature profiles have a maximum inside the gap. If the maximum point (the "hot" point) is only slightly shifted from the average layer ($x = 0.75$) at zero time, it is observed to migrate subsequently to some internal point of the gap ($x = 0.6$), where a temperature breakdown occurs. At that point the velocity gradient and the dissipative heat-release function have maxima. From the shape of the angular velocity profiles, which have a kink, one can conclude that the motion of the liquid from the inner cylinder does not propagate over the whole gap, but only into a certain part of it, and in practice one can identify a motionless zone of liquid (the core) comprising approximately half of the gap. In this case the approximation regarding the average velocity gradient and the use of the isothermal formulas for calculating viscosity become inappropriate.

2. Inner Cylinder Isothermal and Outer Adiabatic: $F = 0$, $\mathrm{Bi} = 0$ (see Fig. 1b). At zero time the temperature distribution is monotonic, and the hot point is located on the outer thermally insulated cylinder. However, a maximum appears later in the temperature profile, and this maximum is displaced inside the gap and stops for $x \approx 0.6$. Thus, in this case there is temperature breakaway (hydrodynamic breakdown) inside the liquid volume. The fact that very favorable conditions for temperature breakaway are created at a certain interior point, and not on the adiabatic surface, is due to the very strong effect of dissipative heat release, which is a maximum for some internal layer having the largest velocity gradient. An increase in temperature of the

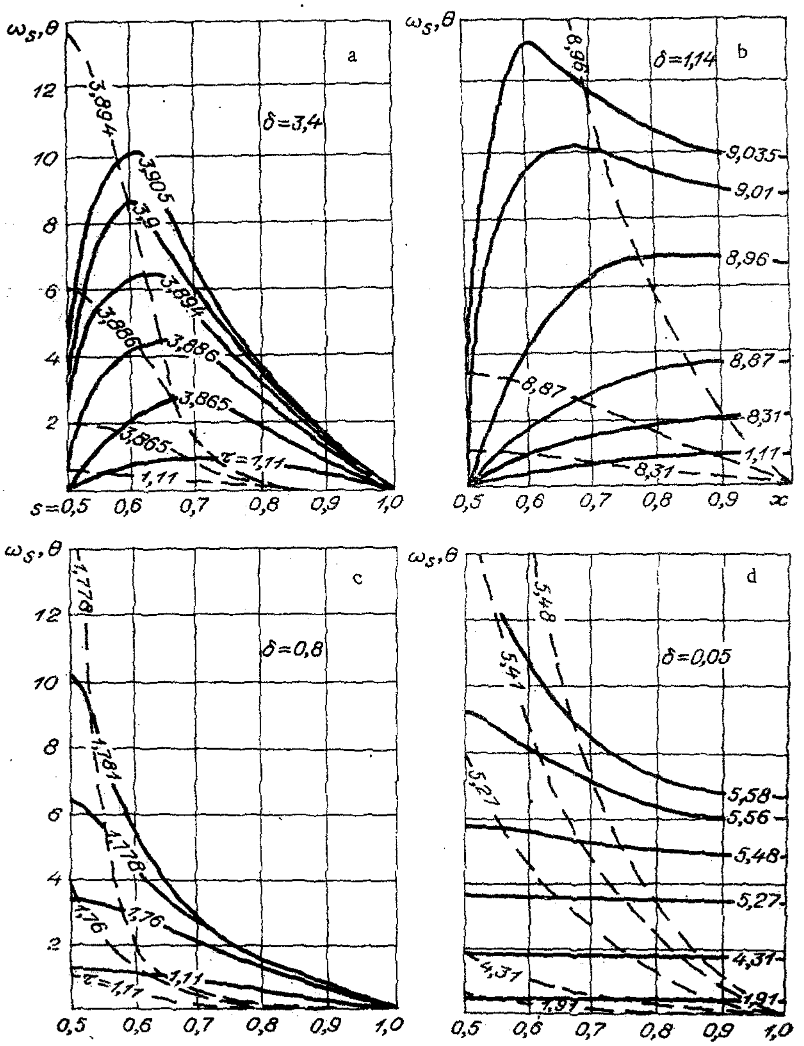

Fig. 1

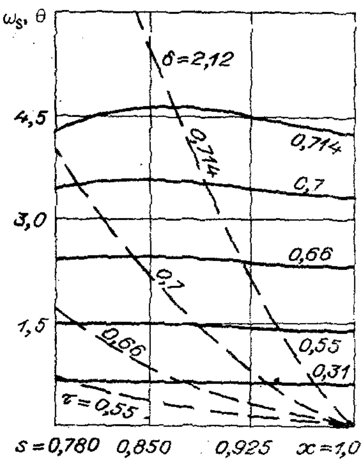

Fig. 2

layers of liquid of the outer cylinder creates conditions for an increase in velocity gradient with time, and, in contrast with the case considered earlier, the liquid motion spans the entire gap.

3. Inner Cylinder Adiabatic and Outer Isothermal: $F = 100$, $Bi = \infty$ (see Fig. 1c). Here the hot point is located on the inner cylinder, where the temperature breakaway also occurs. In this case one can speak of hydrodynamic "ignition" of the liquid from the hot surface of the inner cylinder. Here the motion spans only

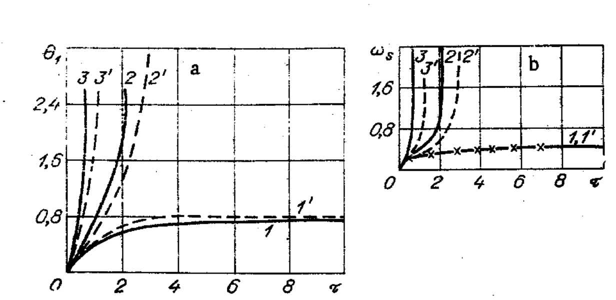

Fig. 3

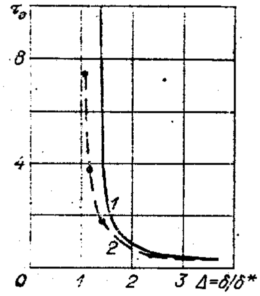

Fig. 4

some inner layer of the liquid, and the layer of liquid adjacent to the outer cylinder remains motionless.

4. Adiabatic Cylinders: F = 100, Bi = 0 (see Fig. 1d). In this case dissipative heating of the liquid occurs uniformly, and the temperature distribution is insignificant up to $\Theta \approx 5$. Then there is weak heating of the inner cylinder (this absence of heating corresponds to $F \to \infty$), and the temperature distribution becomes more significant. The maximum temperature occurs on the surface of the inner cylinder and the hydrodynamic ignition proceeds from there. Since the angular velocity profiles do not have characteristic points in this case, we cannot identify any motionless zone in the liquid.

The absence of heat transfer or the fact that it is small at the cylinder walls is very favorable for viscometric variations in the nonisothermal method [9].

Thus, in all the above cases we can make the following comments:

a) depending on the thermal boundary condition, the inner cylinder experiences either conditions for hydrodynamic ignition (for a hot wall) or for self-ignition when there is temperature breakaway within the liquid volume (for a cold wall); the rules for the development of hydrodynamic thermal breakdown are similar to those for breakdown due to chemical reactions [10];

b) depending on the thermal boundary condition, a wide motionless zone may develop in the liquid on the outer cylinder (for a cold wall), and the heat transmission to this region is determined by heat conduction.

The distributions of temperature and angular velocity across the gap between the cylinders, obtained numerically for the conditions of the experiment with $\delta > \delta^*$, are shown in Fig. 2, which confirms that the above assumption that there is no temperature distribution in the flow zone when conditions are close to adiabatic over a rather wide range of temperature variation is valid [3]. The temperature distribution is insignificant only in strongly heated regions ($\Theta > 4$), which are practically unattainable experimentally. Thus, this is evidence that one can determine the viscosity using the nonisothermal method described in [9].

§3. The experimental investigations of HTB were carried out in a specially developed rotational viscometer with constant momentum and a pneumatic drive [2]. The test material was castor oil ($Pr = 3 \cdot 10^4$). The parameters of the equipment, determined earlier, corresponded to Bi = 2.8, s = 0.8, F = 10 [1]. In [1] there was less heat transfer to the surrounding medium (Bi = 0.7, s = 0.78, F = 10). The coefficient for heat transfer from the liquid to the outer cylinder wall was calculated by the regular regime method [11], and for Bi = 2.8 and Bi = 0.7, it turned out to be $\alpha = 0.483 \cdot 10^{-3}$ and $0.146 \cdot 10^{-3}$ $cal \cdot cm^{-2} \cdot sec^{-1} \cdot deg^{-1}$, respectively.

Figure 3a and b shows a comparison of the experimental and theoretical dependence (dashed and solid lines, respectively) with time of the liquid temperature $\Theta_1(\tau)$ at the wall of the motionless outer cylinder and of the rate of rotation $\omega_s(\tau)$ of the moving cylinder, with Bi = 0.7 (curves 1 and 1' correspond to $\delta = 0.76$, curves 2 and 2' correspond to $\delta = 1.16$, and curves 3 and 3' correspond to $\delta = 2.12$). It can be seen that there is a complete qualitative agreement between the theoretical and experimental results. In the results of the comparison in the region of large rates of strain the structural defects of the instrument are very evident (e.g., defective centering of the rotor), and in the low rate of strain region there is inaccuracy in determining the heat transfer.

It should be noted that for Bi = 0.7 the nature of the curves differs for $\Theta(\tau)$ and $\omega(\tau)$. The liquid warmup $\omega - \tau$ increases monotonically with time, but for the curves θ there is a region of gentle slope, after a sharp initial increase in the rate of rotation (overcoming the inertia of the medium), and only after this is there a progressive increase in ω. The reason is the quasisteady nature of the flow process, in which hydrodynamic variations are caused only by variations in viscosity, which depends exponentially on temperature, according to Eq. (1.1). The basic features of this dependence determine the nature of the function $\omega(\tau)$.

TABLE 1

Method of determination \ Critical parameters	δ^*	Critical temperature			Critical rotation rate	
		T_0^*, °C	ΔT_1^*, °C	Θ_1^*	n^*, rpm	ω_s^*
Experiment	2,27	9,8	12	1,16	410	0,524
Numer. calc.	3,07	13,5	8,53	0,8	500	0,46
Calc. by [1]	—	9,4	10,31	1,0	375	0,483

For Bi = 2.8 (s = 0.8, F = 10) [1] a comparison of the results is shown in Fig. 4, which presents the theoretical curve 1 and the computed curve 2 for the induction period τ_0 as a function of $\Delta = \delta/\delta^*$, the relative distance from the self-ignition limit. For the induction period we assume the time to achieve a prebreakdown warmup of $\Theta = 2$ (i.e., $\Delta T \approx 20°C$). For $\Delta \to 1$ the curves show the greatest discrepancy, and the curves practically merge when the departure from the limit is quite large ($\Delta > 3$).

The critical parameters for hydrodynamic thermal breakdown have been calculated earlier in [1] using an unsteady approach, which did not take into account the radial temperature distribution. Table 1 shows results of the numerical solution calculation of these characteristics in dimensional and dimensionless form and also the critical values of the thermal parameter δ, where T_0^* is the critical thermal breakdown temperature; ΔT_1^*, n^*, Θ_1^*, ω_s^*, are, respectively, the largest values of the steady temperature and rotation rate in dimensional and dimensionless form. A comparison of these characteristics with the experimental values and with the results of the previous computation shows approximate agreement. Thus, it is allowable to compare the critical values of the parameter δ (the analog of the Frank-Kamenetskii parameter in the steady thermal breakdown theory) and the parameter $\varkappa$ from [1] (the analog of the Semenov parameter from unsteady theory) and thus to find the effective heat-transfer coefficient [2].

LITERATURE CITED

1. A. G. Merzhanov, A. P. Posetsel'skii, A. M. Stolin, and A. S. Shteinberg, "Experimental achievement of hydrodynamic thermal breakdown," Dokl. Akad. Nauk SSSR, 210, No. 1 (1973).
2. A. G. Merzhanov, A. M. Stolin, et al., Inventor's Certificate No. 473934; Otk. Izobr. Promysh. Obraztsy, Tovarn Znaki, No. 22 (1975).
3. A. G. Merzhanov and A. M. Stolin, "Thermal theory for viscous liquid flow," Dokl. Akad. Nauk SSSR, 198, No. 6 (1971).
4. S. A. Bostandzhian, A. G. Merzhanov, and N. M. Pruchkina, "Thermal breakdown in viscous liquid flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1968).
5. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "Some problems in nonisothermal steady flow of a viscous liquid," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1965).
6. V. S. Yablonskii and S. A. Kaganov, "Couette flow, allowing for temperature dependence of viscosity and for friction heating," Izv. Vyssh. Uchebn. Zaved., Neft i Gaz, No. 5 (1958).
7. Ya. I Frenkel, Kinetic Theory of Liquids [in Russian], Izd. Akad. Nauk SSSR, Moscow 1059).
8. A. M. Stolin, S. A. Bostandzhiyan, and N. V. Plotnikova, "Critical conditions for hydrodynamic thermal breakdown in flow of a power-law liquid," in: Heat and Mass Transfer. V [in Russian], Vol. 7, Minsk (1976).
9. A. G. Merzhanov, A. M. Stolin, and B. N. Shatalov, A nonisothermal method of investigating the rheological properties of flow systems in the rotary viscometer," in: Heat and Mass Transfer V [in Russian], Vol. 7, Minsk (1976).
10. A. G. Merzhanov, V. G. Abramov, and T. Gontkovskaya, "The rules for transition from self-ignition to hydrodynamic ignition," Dokl. Akad. Nauk SSSR, 148, No. 2 (1963).
11. G. M. Kondrat'ev, Thermal Measurements [in Russian], Mashgiz, Moscow–Leningrad (1957).
12. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics, Plenum Publ. (1969).